Potential games and transportation models

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We consider game theoretic models of the transportation industry, which include models of vehicle use and travel and competition between transportation service providers, additionally incorporating behavioral insights regarding passenger decisions, and focusing on appropriate welfare measures.

We consider a market where the customers are distributed in the vertexes of a transportation graph. The edges of the graph are transportation links (railways, car and air lines, etc.). The vertexes are the hubs (bus stops, airports, railway stations, etc.). The customers are the passengers, who use this kind of transportation. The demand is determined by the flow of passengers. There are n companies (players), who make a service in this market. First, the players form their transportation networks, and then they announce the prices for the service. The consumers choose a service, the use of which leads to the lowest personal costs. The objective of a player (transport company) is to maximize the payoff.

We consider utility function (for passengers) of various kinds, taking into account the price of service, location of stops, time of service etc. Passenger traffic is distributed between competing companies in accordance with the selected utility function. Service companies like to maximise their income.

Knowledge of equilibrium allows us to predict the stable state of a complex system, to which it will come after a certain period of time. Two approaches will be used. To describe the economic equilibrium, the Nash equilibrium will be used, and for the description of the equilibrium of traffic flows - the Wardrop equilibrium.

In 1952 Wardrop hypothesized that any transport system reaches an equilibrium state after some period of time, as well as formulated two principles of equilibrium traffic flows distribution. According to the first principle, the trip time on all existing routes is same for all road users and smaller than the trip time of any road user in the case of its route diversion. The second principle claimed that the average trip time gets minimized.

In many cases, Wardrop equilibria coincide with Nash equilibria as a basic solution concept in non-cooperative game theory. Wardrop's ideas can be further developed by assuming that not only trip time, but also the total costs of road users on all routes are same and minimal. The cost function may include service price, the average trip time, risks and so on.

Transportation model can be considered as a potential game. Potential games were introduced by Monderer and Shapley. In the potential game the equilibrium construction is equivalent to optimization problem in which we need to find the maximum of potential function. We demonstrate the efficiency of this approach in some examples of routing and pricing problems.