## Graphs and collaboration networks

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The online social networks gave impulse to the development of new graph-theoretical methods for network analysis. Furthermore, social network analysis methods are applied in many other fields such as: economics, physics, biology and information technologies.

Betweenness centrality is one of the basic concepts in the analysis of social networks. Initial definition for the betweenness of a node in a graph is based on the fraction of the number of geodesics (shortest paths) between any two nodes that given node lies on, to the total number of the shortest paths connecting these nodes.

We propose new concepts of betweenness centrality for weighted network, current flow centrality, based on Kirchhoff law for electric circuits and game-theoretic centrality measure, based on the concept of Myerson value.

In this approach the nodes of the network can be considered as players. For example, in social network games, nodes are agents (i.e., participants of a social network) and weighted arcs reflect the degrees of their mutual trust or influence. It yields that the prospective tools in this field are the methods of game theory. The main idea here is to find the equilibrium state of the system and then to analyze the rewards of all participants.

In comparison with the traditional measures this new measure can be computed for larger networks. The results of numerical experiments for some examples of networks, in particular, for the popular social and collaboration networks as well as the comparison with PageRank method are presented.

Another important problem here is the community detection in network. The traditional methods for detecting community structure are based on selecting denser subgraphs inside the network. Here we propose to use the methods of cooperative game theory that highlight not only the link density but also the mechanisms of cluster formation. Specifically, we suggest two approaches from cooperative game theory: the first approach is based on the Myerson value, whereas the second approach is based on hedonic games. Both approaches allow to detect clusters with various resolution.