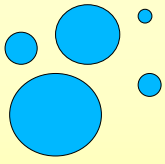
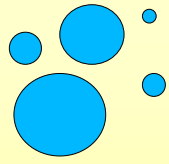
**Math Development 1**

**The building blocks of**

**Early mathematics**

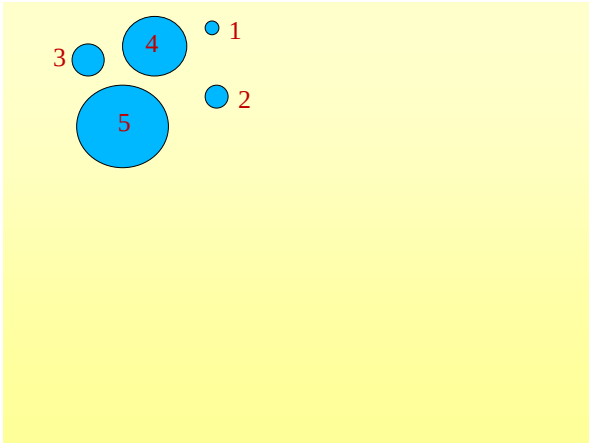




2 or ‘two’ 3

1 or ’one’ 4

5


3
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1
2
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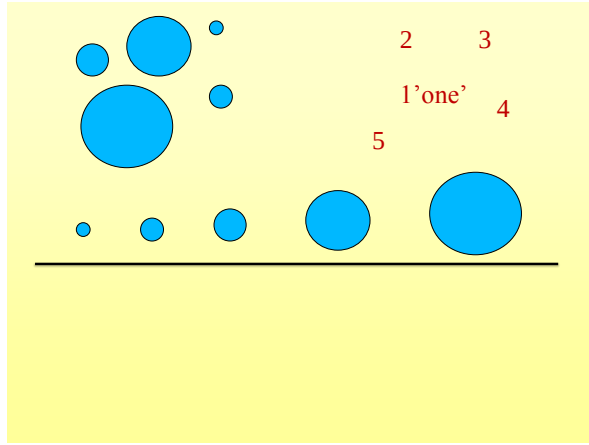
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3
1'one'
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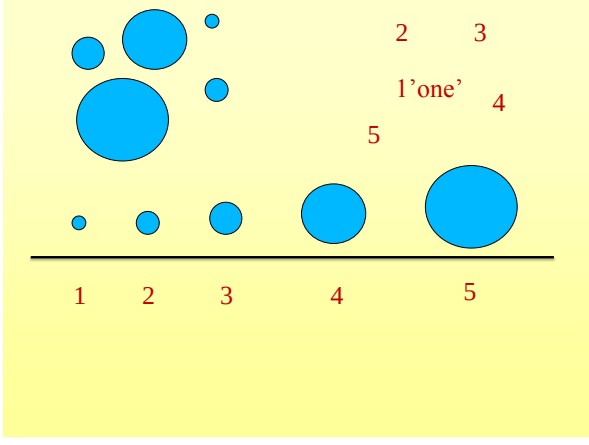
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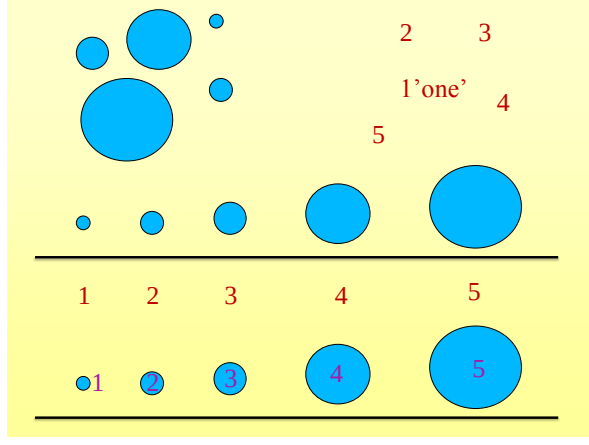
2
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1'one'
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1
2
3
4
5

Number discriminaton in 10-month-old infants

Method: Habituation

1. New stimulus
Infant looks at stimulus

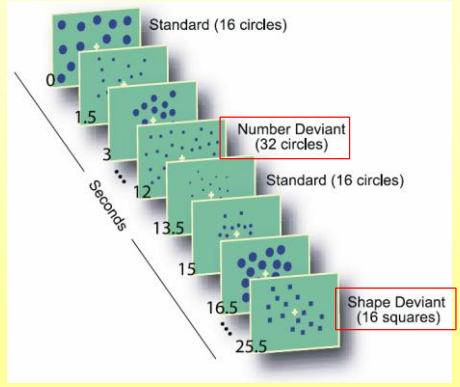
2. Habituation
Infant loses interest in repeatedly shown stimulus

Test

3. Dishabituation
Interest recovers (looking time)

Starkey et al. 1980, *Science*

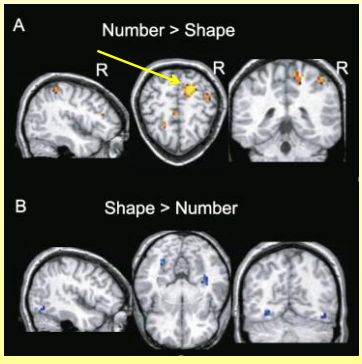
Numerical Change detection Paradigms (fMRI)




Cantlon et al. 2006, PLOS One

Numerical Change detection Paradigms (fMRI)

Number sensitive Areas in 4-year-old children?




Cantlon et al. 2006, PLOS One

The „**number sense**" (intuition for magnitude) is **evolutionarily grounded** (Dehaene, 1997)

Ancient → Cultural

**APPROXIMATE**
Elementary sense (intuition) of magnitude
Evolutionarily grounded (Dehaene 1997)
Biologically primitive ability (precursor)

**Already present in infants And animals?**

**EXACT**
Symbol use (e.g. 1, 2, ..)
Languge of maths
Symbol manipulation

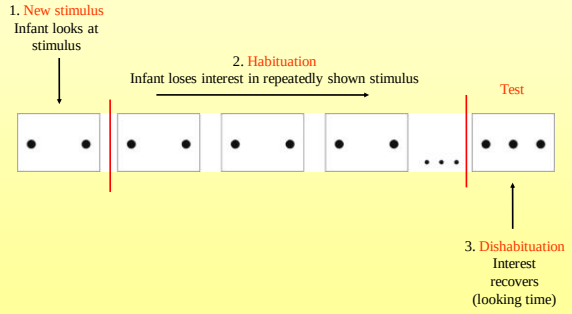
Makes higher maths possible

+

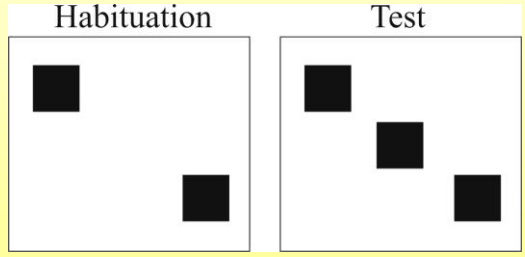
**(HUMAN SPECIFIC) SUPPORTING SKILLS**

Number discriminaton in 10-month-old infants

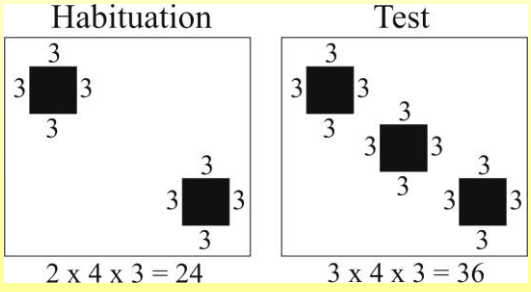
Method: Habituation



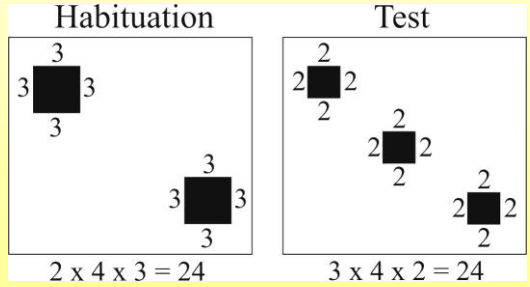

Starkey et al. 1980, *Science*



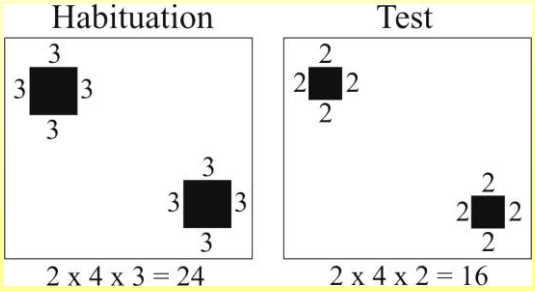

Number changes / visual property changes as well



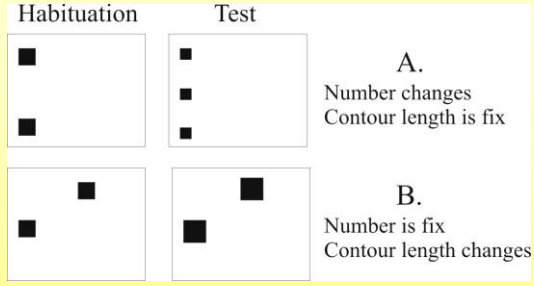

Number changes / visual property fix



Number fix / visual property changes

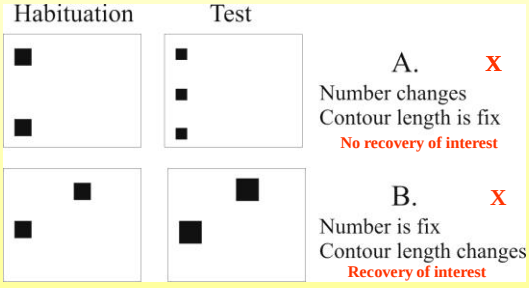



Confounds




Adapted from Clearfield & Mix et al. 1999

Confounds




Adapted from Clearfield & Mix et al. 1999

There are serious visual **confounds** in the stimulus material of most 'number sense' experiments

In addition most 'number sense; studies do not go beyond magnitude perception.

However, even early school maths is much more complex than this...

**What are the building blocks of**

**SCHOOL mathematics?**

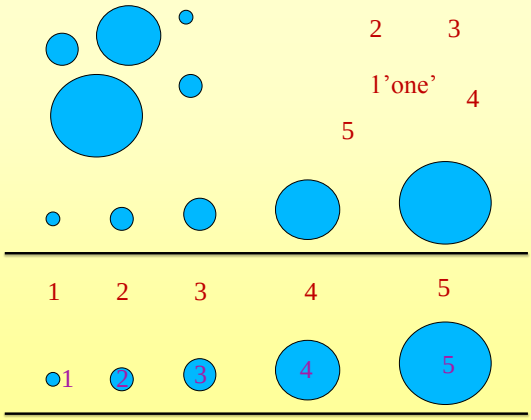
**Children have to acquire a huge skill-set**

**Mathematics is not 'one skill'**

Be empathetic with children('s minds)...

To lots of children, lots of new maths may look as bewildering as probably this looks to lots of you...

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix} \times \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 6 \\ 12 \\ 18 \end{bmatrix}$$




Maths is complex

- **Magnitude/Quantity: Small vs. Large; Few vs. Many, etc.**

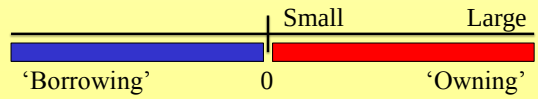
- Associative series: Early **verbal counting** – learning verbal symbols

- Generalized **ordered series**: N, N+1 (conceptual understanding): Digits

- **Associate** quantity with elements of series: 5 < 4 = more < less

- **Number Line**:
  - Visuo-spatial representation of series: positioning numbers on a line
  - Associating magnitude with space
  - 'Dutch' educational tool: The empty number line

- **Extended** number line: zero, negative numbers



## Maths is complex

- **Language**:
  - Numerical facts in **long-term semantic memory** (3×4 ; 2+3)
    Try to memorize something on a similar scale...
    (*Dehaene 1997, Chapter* )
  - Is syntax related to the syntax of maths? (*Varley et al. 2005*)
    3 × ([2×8+4]/4-1×2)=?
- **Rules** (language?)
  - Embedding, preference rules, operation signs [operators] (× + / - =)
- **Concepts**:
  - ZERO: nothing....? Continuum?
  - Number systems: 16 = 20 (octal) = 10 (hexadecimal) = 10000 (binary)
  - apple + orange; eat it... ; - orange = apple
  - A+B-B = A ; A+B-A=B (**abstract** level is harder...)

## Maths is complex

-**Visuo-spatial** abilities?
  - Place Value: 1.2 ; 100.2 ; 0.2 ; 0.0002 ; 1E2 ; 1E-2

    347 > 30047?

    This is also another kind of arithmetic syntax.
    (*McCloskey 1992*)
  - Operation **techniques**: spatial manipulation

```
  1009
+ 2345
---------
  3354
```

## Maths is complex

- **Memory** (*Geary* )
  - **Phonological memory**: keep operators and operands in mind
    If you forget/confuse details, outcome will be wrong even if you know the rule...
  - **Visual short-term memory**: e.g. Large number addition, etc.
  - **Working memory**: parallel operations: e.g. Keeping partial results in mind while computing another one

```
  1867
+ 2399
---------
  4266
```

-- **Do** 6+9 **AND memorize** '1' **WHILE** remembering to add the OTHER '1' (from 7+9) to 15 what has **also** been **memorized** (at the same time: follow the **visual** pattern...

*Multi-step problems: carrying, borrowing...*
*24+18=?*

## Maths is complex

- **Part and whole relations**
- **Fractions**: There are numbers between 'numbers' in the number line
  - strange operations: e.g. Fraction division:
    2/4 / 1/4 = 2/4 x 4/1 = 8/4 = 2
- **Story problems:**
  - Translate an everyday problem into an abstract representation...

There were 10 books on the bottom shelf, 30 books on the middle shelf and three books on the top shelf. How many books would be on each shelf if they were all shared out evenly?

**Huge memory load!**

## Maths is complex

- **Attention**: staying focused, knowing where you are, selecting the right step

- **Inhibition**: Resisting distraction (from classmates, irrelevant parameters, e.g. Nice figure on the side...)

- **General processing speed**: Maths is so complex that if you slow down at Some points you may e.g. forget partial results, operands, etc.

## Maths is complex, precise, step-by-step and abstract

- **Lots of skills and representations** to use to reach a result

- You have to use them in concert and one after the other (**coordinate** a PLAN!!!)

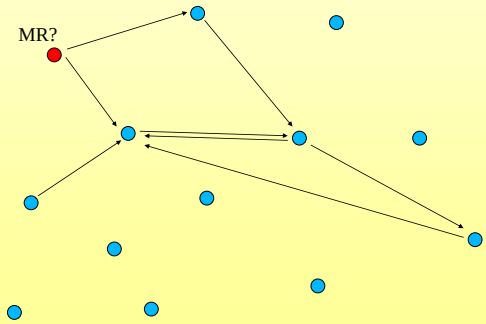
- Usually there is only **ONE** good outcome *(the **Highlander** also has a hard job...)* (e.g. You can write an essay in multiple ways but there is only one [or very few] correct solution[s] in most maths problems

- Requires **abstraction, a new 'language'** (everyday language is not enough...)

>> Maths seems to be really more **demanding** than other subjects

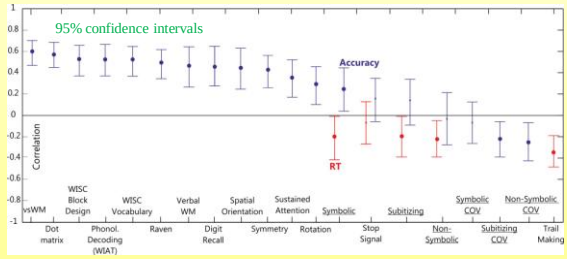
- **Maths Anxiety**: Emotional factors

## Maths is complex, precise, step-by-step and abstract
No single variable is able to explain maths performance
A co-ordinated network of skills is necessary




**Modelling** numerical competence (**N=98** 9-year-olds)
(Bootstrap correlations; 100,000 permutations)

**Correlation (Y axis) between math score and cognitive scores**




Szucs et al. *Developmental Science*, 2014

Modelling numerical competence (N=98; 9-year-olds)
(Bootstrapped BETA confidence intervals, 100,000 permutations)

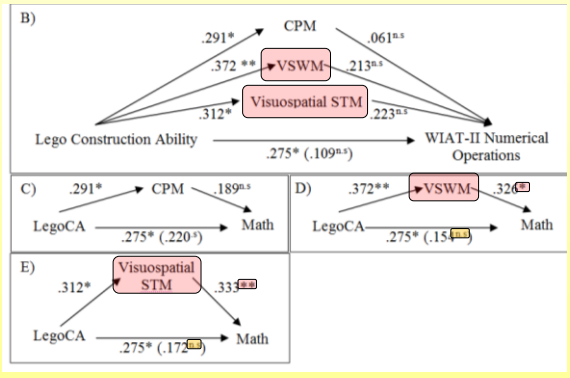

Szucs et al. *Developmental Science*, 2014

## Visuo-spatial STM/WM in the classroom?
## 7 year-olds

- WIAT – II
  - Numerical and Reading
- Raven's Children's Progressive Matrices
- Automated Working Memory Assessment

| | Verbal | Visuospatial |
|---|---|---|
| WM | Listening recall | Odd One Out |
| STM | Digit recall | Dot Matrix |

Nath & Szucs, *Learning and Instruction*; 2014

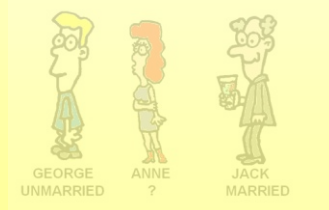



Nath & Szucs, *Learning and Instruction*; 2014

## **Logical reasoning** in DD and in gifted children

Jack is looking at Anne, but Anne is looking at George.
Jack is married but George is not.
Is a married person looking at an unmarried person?

a) yes
b) no
c) cannot be determined




Morsanyi,…, Szucs, 2013; *Developmental Science*

## Logical reasoning in DD and in gifted children

16 transitive inference problems with the following structures:

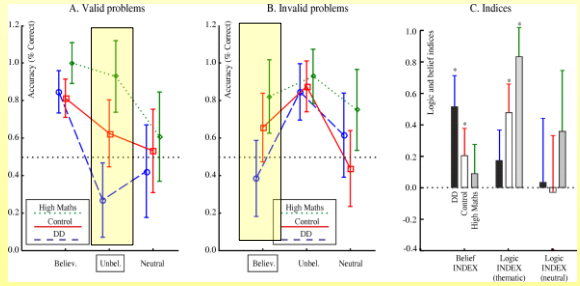
1. A>B, B>C A>C? (**valid**, easy structure)
2. A>B, C>A C>B? (**valid**, difficult structure)
3. A>B, B>C C>A? (**invalid**, easy structure)
4. A>B, C>A B>C? (**invalid**, difficult structure)

• 8 **belief-laden** problems:
  – 4 with **believable** conclusions (e.g., *elephants are bigger than mice*)
  – 4 with **unbelievable** conclusions (e.g., *rabbits are stronger than gorillas*)

• 8 **belief-neutral** problems (neither believable nor unbelievable):
  – 4 **visual-spatial (**e.g., *the panda is behind the giraffe*)
  – 4 **non-visual** (e.g., *Sarah is older than Anne*)

Morsanyi,…, Szucs, 2013; *Developmental Science*

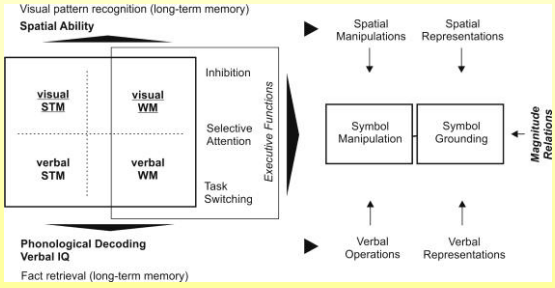
## Logical reasoning in DD and in gifted children




**Role of visuo-spatial memory and inhibiton ability in reasoning?**

Morsanyi,…, Szucs, 2013; *Developmental Science*

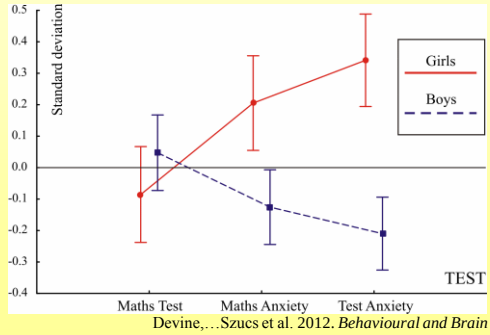
## Modelling numerical competence (N=98; 9-year-olds)




Szucs et al. *Developmental Science*, 2014

## Emotional factors: Mathematics anxiety

433 children in the UK; School Years 7,8 and 10




Devine,…Szucs et al. 2012. *Behavioural and Brain functions*

**What is important in math development?**

- Strong proposals about ‘number sense’
  - However, most designs are **confounded** by non relevant visual variables

- Maths requires a very **complex** set of skills

- There are **several** alternative suggestions
  Verbal/visual memory / inhibition / phonology /
  Spatial skills / control skills

- Here we focused on **visuo-spatial memory**, a particularly strong correlate of math development

- Emotional factors matter as well; e.g. **Math anxiety**